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ISLAMIC STOCK MARKET: Modeling Volatility and Comparative Study

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Abstract: This article aims to discuss a comparative approach between the Islamic and a conventional index. First, we analyzed some empirical properties of the Islamic and a conventional daily returns index, we noticed that they capture most stylized facts observed in financial markets, namely the serial correlation, leptokurticity and heteroscedasticity effect. As we detected heteroscedasticity effect, we modeled the returns both of the Islamic Sharia SP500 and its conventional counterpart, and we analyzed the persistence of their volatility using the GARCH model. The results show that volatility persistence of both indexes was very significant and the **S&P** 500 index Shariah was less volatile than the conventional index at a long run and it does present less risk at crisis periods.

Key words: S&P Sharia, daily returns, conditional volatility, financial crisis, GARCH models.

INTRODUCTION

The financial dynamic market is among the most complicated economic phenomena. Indeed, uncertainty controls the financial assets evolution. Modeling this random dynamic evolution is obviously a complicated task knowing that many factors influence the financial markets including the massive intervention of investors and the manager's funds. Also all related information on economic conditions is deterministic and may be a major component for making financial decision. (Herlin, 2010).

The French mathematician Bachelier (1900) introduces mathematical modeling of price movements and evaluation of contingent claims in financial markets. His idea consists of representing the prices evolution by a probability distribution. Thus, the specific properties of the Gaussian model will facilitate works designed for modeling financial series, including the Black-Scholes model which help to the option pricing for stock and highlighting their stochastic nature. However, existing models were not able to prevent the financial crises occurrence whose frequency has increased in the early 21th century, namely the 2007-2008 financial crises (Herlin 2010). Thus, the important crisis frequency raises questions about the reliability of models developed and their basic assumption relevance.

Islamic finance is among the alternatives to conventional finance. In fact, modern Islamic finance began to develop in the early 1970s. It differs from conventional finance by its conception of the value of capital and labor.

The basic principle of Islamic finance is the prohibition of all forms of interest (Riba). The other principle is the equal sharing of profits and investment risks. These profits must be generated by investments in real assets and through a fair and legitimate trade. Any Investment in companies involved in activities related to alcohol, tobacco, gambling or pornography is strictly prohibited.

In summary, the Islamic financial model is based on five main pillars, which are:

1. The prohibition of Riba (usury);
2. The prohibition of gharar (speculation) and Maysir (uncertainty);
3. The need for investment in legitimate sectors;
4. The obligation to share profits and losses;
5. The backing of tangible assets investment in the real economy.

The first Islamic index was launched on the market in 1998. It is the “Index Socially Aware Muslim” SAMI.

Since then, the major index providers have expanded their traditional range and they offer now a wide range of Sharia clues to accompany the rapid growth of Islamic Finance, particularly the funds “Sharia Compliant”.

Through this array of signs Sharia, all geographic areas, all sectors and all levels of capitalization are covered. There are also indices representing the Sukuk market. The majority of Islamic indexes are subsets of benchmarks in the sense that their construction is the result of filtering their parent index. Islamic indices use different screening methods of security in their selection (Hashim, 2008).

The purpose of the following article is to conduct a comparative approach between two stock market indexes S$P Sharia and its conventional counterpart S$P500 in terms of volatility behaviors. Before considering such a comparative study, we are going to exhibit some statistical properties of returns Islamic index in order to see if they capture most of the stylized facts observed in financial time series.

In fact, empirical studies have analyzed the stochastic behavior of financial series and summarized it in the form of stylized facts, including papers of Cont (2001) and Swell (2011) describing some common properties to most conventional financial series; including serial correlation, asymmetry and leptokurticity and the heteroscedasticity effect of the return distribution. We will first see if those properties are satisfied for an Islamic index and how it differs statistically from its conventional counterpart. Then, we conduct our comparative approach in terms of persistence’s volatility.

1. DATA AND DESCRIPTIVE STATISTICS

1.1. Data

We will highlight some statistical properties observed through the empirical study of the Islamic market index SP 500 Shariah evolution during the period from 29 December 2006 to 07 March 2011.This period involved the subprime crisis of 2007. In fact, the past decade has been the worst in the financial markets history in terms of amounts of losses that threatened to disintegrate the global financial system. Also, this incites debate on some alternative mathematical models able to be near form financial market reality.

Throughout this article, we adopt the following notations:

S_t : The index course at time t

r_t : The corresponding logarithmic return that refers to the relative change in stock prices:

$$r_t = \ln S_t - \ln S_{t-1} = \ln \frac{S_t}{S_{t-1}} = \ln(1 + X_t),\ X_t = \frac{(S_t - S_{t-1})}{S_{t-1}}$$

1.2. Descriptive statistics

We are going to plot the distribution of closing and returns values to describe their evolution on the time.

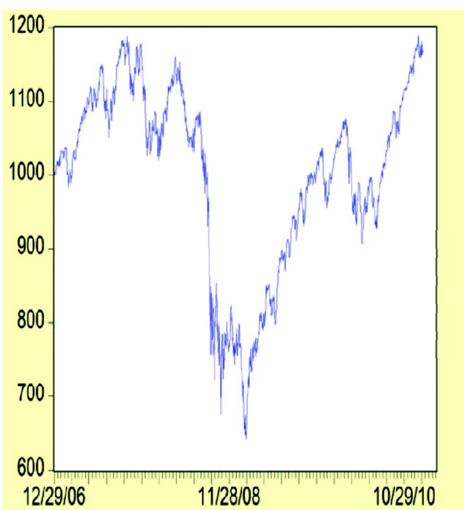

Figure 1: Closing value of SP sharia index

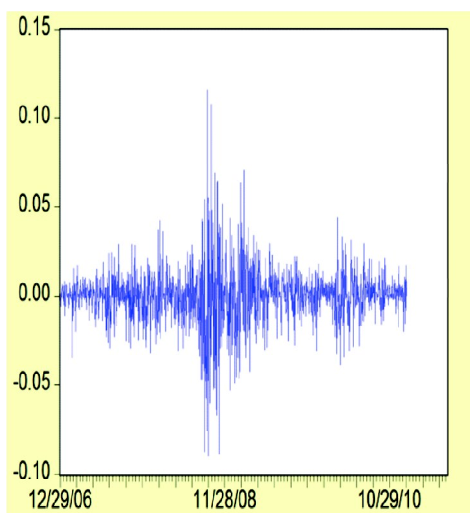

Figure 2: Returns value of SP sharia index

It's clear from figure 1 that SP sharia closed values were increasing since 2006 until they suddenly declined in a dramatic way at the subprime crisis of 2007.

However, they had started to rise again by the beginning of 2009, but the level or recovery is still low in comparison with the index level just before the crisis. The stationarity of the SP sharia closed values is not verified. The stationnarity will be studied for the first level differentiation of closing values.

Thus, the Islamic daily returns index appears stationary around a constant (figure 2). We observe that fluctuations take both negative and positive values around the mean. Moreover, the returns plot of the S$P Sharia exhibits negative peaks in 2008 and 2009. The following table presents some descriptive statistics of both the Islamic daily returns and theconventional indexes.

Table 1
Statistical properties of both return indexes

	S$P Shariah	*S$P 500*
Number of observations	1053	1053
Mean	0.000263	0.000077
Median	0.001016	0.000898
Maximun	0.122799	0.115800
Minimum	-0.090855	-0.090350
Std.Dev	0.015477	0.016985
Skewness	0.263343	0.021419
Kurtosis	12.64286	10.37128
Jarque Bera	4091.876	2384.065
Probability	0.000000	0.000000

We can see from the table 1 that the returns average of the Islamic index exceeds that of the conventional index. They obtained respectively 0.03% and 0.008%.

Also, the Islamic index is less volatile (Std.Dev = 0.15%) than the conventional index (Std.Dev = 0.17%) . This means that the conventional index fluctuated more than the Islamic index because of the financial crisis that generated great losses to the banking sector in 2007.

Both the returns are slightly asymmetric with a positive skewness. Their kurtosis is largly greater than 3 of the normal distribution.

The high value of Jarque Bera combined with skewness and kurtosis values show that the return distributions of both indices don't follow a normal law.

We are going to analyze the Islamic serial correlation index. For the conventional index, many empirical studies had proved that SP 500 index capture all the stylized facts observed in the financial markets (Pochard 2006). Thus, we consider that the daily returns autocorrelation of SP500 index is low, but the squared daily returns autocorrelation of SP500 is significant. We are going to verify if this property is confirmed for the Islamic daily returns.

Table 2
Returns autocorrelation of SP sharia index

	*AC**	*PAC**	*Q-Stat**	*Prob*
1	-0.147	-0.147	22.889	0.000
2	-0.124	-0.148	39.045	0.000
3	0.122	0.083	54.852	0.000
4	-0.042	-0.029	56.750	0.000
5	-0.024	-0.008	57.350	0.000
6	0.029	0.006	58.267	0.000
7	-0.047	-0.041	60.614	0.000
8	0.025	0.019	61.278	0.000
9	0.009	0.001	61.364	0.000
10	0.023	0.041	61.921	0.000
11	-0.027	-0.024	62.680	0.000
12	0.067	0.070	67.468	0.000
13	-0.004	0.007	67.483	0.000
14	-0.048	-0.028	69.952	0.000
15	-0.068	-0.096	74.860	0.000
16	0.080	0.052	81.736	0.000
17	-0.036	-0.025	83.117	0.000
18	-0.086	-0.076	91.142	0.000
19	0.052	0.009	94.025	0.000
20	0.050	0.047	96.745	0.000

Table 3
Squared returns autocorrelation of SP sharia index

	AC	*PAC*	*Q-Stat*	*Prob*
1	0.161	0.161	207.72	0.000
2	0.254	0.235	726.44	0.000
3	0.131	0.067	864.27	0.000
4	0.117	0.037	973.18	0.000
5	0.211	0.162	1331.1	0.000
6	0.123	0.048	1451.7	0.000
7	0.116	0.010	1558.9	0.000
8	0.114	0.042	1663.0	0.000
9	0.125	0.062	1788.3	0.000
10	0.098	0.007	1864.8	0.000
11	0.142	0.067	2027.6	0.000
12	0.099	0.028	2105.5	0.000
13	0.086	-0.004	2165.1	0.000
14	0.054	-0.028	2188.4	0.000
15	0.087	0.034	2249.2	0.000
16	0.092	0.031	2317.6	0.000
17	0.091	0.021	2384.8	0.000
18	0.096	0.032	2459.3	0.000
19	0.078	0.019	2507.9	0.000
20	0.078	0.004	2556.3	0.000

AC* : Sample autocorrelation
PAC* : the partial correlation between two variables is the amount of correlation between them which is not explained by their mutual correlations with a specified set of other variables.
Q-Stat* : The q statistic or studentized range statistic is a statistic used for multiple significance testing across a number of means.

According to those tables, the daily returns autocorrelation are low; but the squared returns autocorrelation are significant .Since all the Q-stat values are largely greater than 3.84; we reject the hypothesis of return independence (white noise hypothesis). This hypothesis refers to the notion of market efficiency. Under this assumption, the price of a stock incorporates all relevant information (Fama, 1965). The hypothesis of market efficiency means that prices can vary between t and t +1 due to the arrival of unanticipated news. This means that after a price increased yesterday,it is almost as likely to observe a rise or fall in prices today. For the trader or investor, this means that it is difficult to use the information of past prices to predict future prices. This hypothesis is not verified in our case.

Thus, the process of logarithmic returns seems effectively not self-correlated; however it is not the same thing for the squared returns. In fact, the autocorrelation function of squared logarithmic returns remains significantly positive for a long time and thus indicates the existence of dependence between logarithmic returns. This fact is called the long memory which means that a shock at a time t has an impact on future returns. So, there is an interdependence of daily returns that persists for a long time. In our case, we have obviously shown that the financial crisis have influenced significantly the Islamic index returns (figures 1 and 2).

In summary, the Islamic index is characterized by the same statistical properties common to most financial series. Thus, the distribution of Islamic daily returns index is leptokurtic and asymmetric unlike a normal distribution. We also noticed that there is no autocorrelation of returns but there is presence of squared returns autocorrelation. Thus we reject the hypothesis of white noise, which states that returns are independent and identically distributed. The presence of long memory in financial markets is also proved; this concept identifies a character for a long time of shocks and its impact on future returns. Moreover, and according to figure 2, we noticed extreme fluctuations in Islamic return index due to the financial crisis that reflect an important variability in stock market volatility. So, the volatility is not constant and depends on time. The ARMA models are not able to explain the relevant features of financial data. For this reason, we are going to test the heterosceadasticity effect. Before, we are going to develop some methods of volatility measures.

2. VOLATILITY MEASURES

2.1 Literature reviews

Models for volatility and forecasting volatility stock market have been the subject to vast empirical and theoretical investigations over the past decade by academics and practitioners. There are a number of motivations for this line of inquiry. Arguably, volatility is one of the most important concepts in finance. Volatility, as measured by the standard deviation or variance of returns, is often used as a basic measure of the total risk of financial assets. Many value-at-risk models require the estimation or forecast of a volatility parameter. Also, the volatility of stock market prices enters directly into the Black— Scholes formula for deriving the prices of traded options (Brooks, 2008).

The simplest model for volatility is the historical estimate. Historical volatility simply involves calculating the variance of returns in the usual way over some historical period that becomes the volatility forecast for all future periods. The historical average variance was traditionally used as the volatility input to options pricing models. Although there is a growing body of evidence suggesting that the use of volatility predicted from more sophisticated time series models will lead to more accurate option valuations (Akgiray, 1989; Chu and Freund, 1996). Historical volatility is still useful as a benchmark for comparing the forecasting ability of complex time models.

One of the historical volatility extensions is the exponentially weighted moving average (EWMA) which allows more recent observations to have a stronger impact on the forecast of volatility than older data points. Under a EWMA specification, the latest observation carries the largest weight, and weights associated with previous observations decline exponentially over time. This approach has two advantages over the simple historical model. First, volatility is in practice likely to be more affected by recent events. Second, the effect on volatility of a single given observation declines at an exponential rate as weights attached to recent events go down. On the other hand, the simple historical approach could lead to an abrupt change in volatility once the shock falls out of the measurement sample. And if the shock is still included in a relatively long measurement sample period, then an abnormally large observation will imply that the forecast will remain at an artificially high level even if the market is subsequently calm.

On the other hand, the implied volatility is the market's forecast of the volatility of underlying asset returns over the lifetime of the option. In fact, all pricing models for financial options require a volatility estimate or forecast as an input. Given the price of a traded option obtained from transactions data, it is possible to determine the volatility forecast over the lifetime of the option implied by the option's valuation. For example, if the standard Black-Scholes model is used, the option price, the time to maturity, the risk-free rate of interest, the strike price and the current value of the underlying asset, are all either specified in details of the options contracts or are available from market data. Therefore, given all of these quantities, it is possible to use a numerical procedure, such as the method of bisections or Newton-Raphson to derive the volatility implied by the option (Watsham and Parramore, 2004).

Autoregressive volatility models are a relatively simple example from the class of stochastic volatility specifications. The idea is that a time series of observations on some volatility proxy are obtained. The standard Box-Jenkins-type procedures for estimating autoregressive (or ARMA) models can then be applied to those series. If the quantity of interest in the study is a daily volatility estimate, two natural proxies have been employed in the literature: squared daily returns, and daily range estimators. Producing a series of daily squared returns trivially involves taking a column of observed returns and their squared values. The squared return at each point in time t becomes the daily volatility estimate for day t. A range estimator typically involves calculating the log of the ratio of the highest observed price to the lowest observed price for trading day t, which then becomes the volatility estimate for day t. (Brooks 2008).

Taking into account the imperfections of recent measures, the volatility measures used in our paper is based on an Autoregressive Conditional Heteroscedasticity model (ARCH) (Engle, 1982). Furthermore, other method of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) (Bollerslev, 1986) will also be examined. These models are adopted since they allow the heteroscedasticity in residual series. Furthermore, these models take into account the volatility shock to persist over time (Ibrahim, 2002).

The GARCH (p, q) model is :

$$r_t = \varepsilon_t \sigma_t \tag{1}$$

$$\sigma_t^2 = \alpha_0 + \Sigma_i^p \alpha_i \, r_{t-i}^2 + \Sigma_j^q \beta_j \, \sigma_{t-j}^2 \tag{2}$$

Equation 1 represents the conditional which is modeled as an autoregressive process. The ε_t is selected such that the residuals are not serially correlated. Equation 2 refers to the conditional variance. The conditional variance σ_t^2, depends on lagged squared errors and lagged conditional variances. In order to be well defined, GARCH model necessitates that coefficient of the lagged squared errors and lagged conditional variances to be non-negative and their sum mst be less than unity.

2.2 Empirical results

We will ensure the non-homoscedastic or heteroscedasticity in the residual presented in the model by testing it with the ARCH LM Test.

The results for both Islamic and conventional test index are respectively:

Table 4
ARCH LM test for Islamic index

F-statistic	31.629	Probability	0.000
Obs*R-squared	30.762	Probability	0.000

Table 5
ARCH LM test for conventional index

F-statistic	31.169	Probability	0.000
Obs*R-squared	30.328	Probability	0.000

We recall that the H_0 hypothesis is: the homoscedasticity exists.

We noticed that the probability of Obs*R-squared = 0.00 <0.05, so we reject the H_0 hypothesis for both Islamic and conventional indexes. Thus, there is heteroscedasticity effect for both indexes.

Now, we are going to estimate conditional volatility parameters of GARCH (1.1).

Table 6
Conditional volatility parameters estimated with a GARCH (1.1) Model

	α_0	α_1	β_1	σ^2
SP SHARIAH	3.19E-06	0.093669	0.887829	0,0001724
SP 500	2.88E-06	0.086306	0.900585	0,0002197

We noticed that the conditions $\alpha_1 > 0$, $\beta_1 > 0$ and $\alpha_1 + \beta_1 < 1$ are verified.

The first term α_0 corresponds to the minimum variance threshold below which the conditional variance does not go down. For the Sharia index, it's slightly higher compared to the conventional index.

α_1 is a sum of squared residuals, which reflects the impact of shocks on volatility. When a shock occurs at time t, the return's value is very different from its mean and so the residue is very large. Its value is slightly higher for the Islamic index. Hence the Islamic index was affected by shocks to the financial crisis of 2007 with a slightly greater extent relative to its conventional counterpart.

The last term β_1 matches the sum of recent variances which reflect the volatility persistence. Here, we note that the volatility is highly persistent for both the Islamic and conventional indexes with slight difference between.

Finally, unconditional volatility of Shariah index is also slightly larger than the conventional index. This confirms that the persistence of the volatility of the conventional index remains significant in the long run due to the financial crisis.

The Islamic daily returns index captures most of the stylized facts observed in financial markets. We showed that the daily returns of SP sharia are asymmetric and leptokurtic, unlike a Gaussian distribution. Leptokurticity reflect that the distribution tails are relatively thicker than those of a Gaussian distribution. Moreover, the extreme fluctuations due to financial crisis generate important variability of volatility which depends on the recent volatilities and recent squared residuals. We also proved absence of daily returns autocorrelation for the SP sharia, but the daily squared returns autocorrelation are significant. Thus, there is presence of long memory which reflects that a shock at the time t has an impact for a long run on the futures returns values. So, the white noise hypothesis is not accepted, hence, we can conclude that the daily returns of SP sharia are not independent and identically distributed. We also proved presence of heteroscdeasticity effect for both Islamic and conventional indexes. We modeled the conditional variance for both Islamic and daily returns indexes using ARCH models. The results of modeling and estimating parameters showed that there is significant persistence volatility for both indexes, and we noticed that Islamic index is less volatile on the long run than its conventional counterpart.

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